



## Electrical Power Measurements

### The Measurement of Electrical Power

With today's emphasis on conserving electrical energy, many test measurements are required to first determine consumption before deciding what can be done to reduce it. Unfortunately, there are many areas of measurement where much has been taken for granted for a long period of time, so proper instructions and guidelines for conducting these tests are difficult to find. One such area is the measurement of electrical energy in industry. It is hoped that this discussion will aid those who must make power measurements of this type.

### Measuring Power in AC Circuits

Power measurements in AC circuits present more difficult measurement problems, even more so in polyphase circuits. Since the flow of current in an AC circuit constantly changes direction (alternating current) due to the alternating polarity of the voltage, the power in an AC circuit is not as easy to measure as in a DC circuit. In a purely resistive circuit, the voltage and current relationship would be as shown in Figure 1 and power measurements could simply be made by measuring the RMS current and voltage and applying the formula  $P=EI$ .

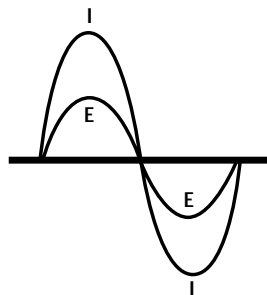


Fig. 1

However, AC circuits are seldom purely resistive since there is usually some reactance (impedance) present (inductive or capacitive). When reactance is present, it influences the relationship of the voltage to the current in the circuit so that true power is no longer represented by  $P=EI$ . Figure 2 shows voltage and current in a circuit whose load is both resistive and inductive.

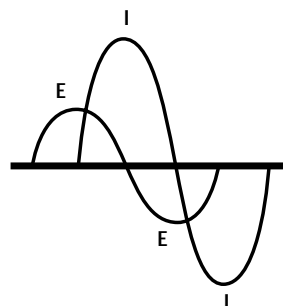


Fig. 2

The formula  $P=EI$  can no longer be applied since  $E$  and  $I$  are "out of phase" with each other and this phase difference must be accounted for. This phase difference is called Phase Angle ( $\theta$ ),

### Phase Angle

Phase Angle is defined as the difference in degrees by which current leads voltage in a capacitive circuit or lags voltage in an inductive circuit. In series circuits, it is equal to the angle whose tangent is given by the ratio  $X \div R$  and is expressed by the arc  $\tan X \div R$ , where  $X$  = the inductive or capacitive reactance in ohms and  $R$  = the non-reactive resistance in ohms of the combined resistive and reactive components of the circuit under consideration.

Therefore,

in a purely resistive circuit  $\theta = 0^\circ$

in a purely reactive circuit  $\theta = 90^\circ$

also when

$\theta = 0^\circ$ ,  $\cos \theta = 1$ , and  $P = EI$

$\theta = 90^\circ$ ,  $\cos \theta = 0$ , and  $P = 0$ .

When there is a phase angle difference between voltage and current, the formula for true power becomes  $P = EI \cos \theta$ .  $\cos \theta$  is called the "Power Factor" which is created by this phase difference.

### Power Factor

The power factor of any AC circuit is equal to the power in watts divided by the apparent power in volt-amperes which is equal to the cosine of the phase angle and is expressed by

$$\text{p.f.} = \frac{EI \cos \theta}{EI} = \cos \theta$$

where

p.f. = the circuit load power factor

$EI \cos \theta$  = the true power in watts

$EI$  = the apparent power in volt-amperes

$E$  = the applied potential in volts

$I$  = the load current in amperes

Therefore,

in a purely resistive circuit  $\theta = 0^\circ$  and p.f. = 1

and in a reactive circuit,  $\theta = 90^\circ$  and p.f. = 0

A summary of formulas for power measurement in DC and AC circuits follows:

### Standard Formulas for Power Measurement

DC Circuits	AC Circuits
$P = EI$	$P = EI \cos \theta$
$P = E^2 \div R$	$P = E^2 \cos \theta \div Z$
$P = I^2 R$	$P = IZ \cos \theta$
	$P = EI \text{ p.f.}$

$Z$  = absolute value of impedance in ohms

Fortunately, there are instruments which take into account all factors, current, voltage, and power factor at the same time and indicate the true power being consumed. These are available in both analog and digital configurations.

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### Single and Polyphase Circuits

Single-phase power is the simplest to measure as shown in Fig. 3. While many of the loads throughout a plant are single-phase, their operating power comes from one phase of a polyphase distribution system. Heavy electrical loads, such as large motors, are generally of the three-phase type so it is necessary, therefore, that power measuring equipment is capable of handling polyphase circuits.

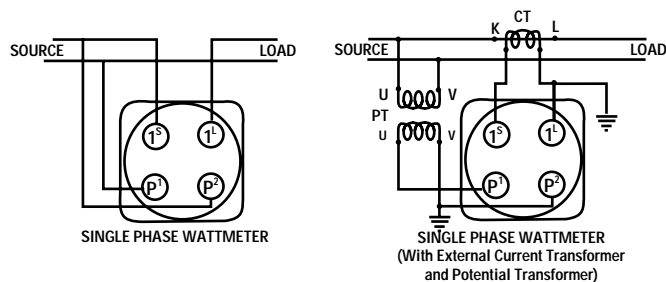


Fig. 3

### Blondel's Theorem

Blondel's Theorem states that if a network is supplied through  $N$  conductors, the total power is measured by summing the readings of  $N$  wattmeters so arranged that a current element of a wattmeter is in each line and the corresponding voltage element is connected between that line and a common point. If the common point is located on one of the lines, then the power may be measured by  $N - 1$  wattmeters.

This means a 3 phase, 3-wire system requires only two single-phase wattmeters or one polyphase instrument with two measuring elements. Four-wire circuits require three wattmeters or a three-element instrument. Space does not permit proof of this theorem, but it does give true power readings under unbalanced conditions.

### Typical Polyphase Circuit Connections

A 2 phase 4-wire circuit (not interconnected) may be treated as equivalent to two single-phased circuits. Two wattmeters are connected as shown in Fig. 4; total power is the arithmetical sum of the two instrument readings.

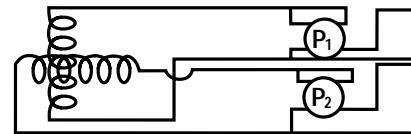


Fig. 4 Power in 2-phase 4-wire circuit (not interconnected)

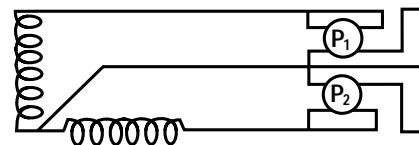


Fig. 5 Power in 2-phase 3-wire circuit

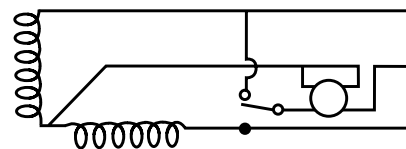


Fig. 6 Power in 2-phase 3-wire circuit, one wattmeter

A 2-phase 3-wire circuit requires two wattmeters connected as shown in Fig. 5; total power is the algebraic sum of the two readings. This connection is correct for any of load and power factor.

One wattmeter may be used, connected as shown in Fig. 6 if there is no load across the outer conductors and the phases are balanced as to load and power factor. Readings are summed for the two switch positions.

A 2-phase 4-wire interconnected circuit requires three wattmeters, connected as shown in Fig. 7; total power is the algebraic sum of the three readings. This connection is correct under all conditions of load and power factor. It will be noted that the voltage impressed on  $P_1$  is times the voltage on  $P_2$  and  $P_3$ . Two wattmeters, one in each phase, will give the power only when the load is balanced in all four legs.



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### Circuit Connections (*Continued*)

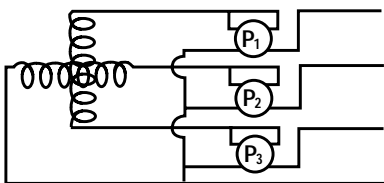


Fig. 7 Power in 2-phase 4-wire interconnected circuit.

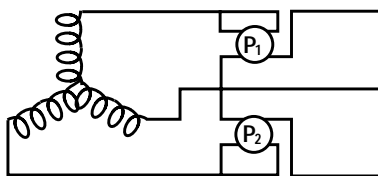


Fig. 8 Power in 3-phase 3-wire circuit, 2 wattmeters.

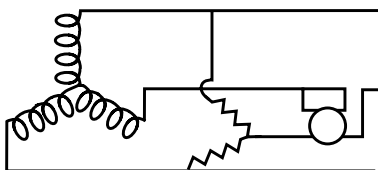


Fig. 9 Power in 3-phase 3-wire circuit, one wattmeter with Y box.

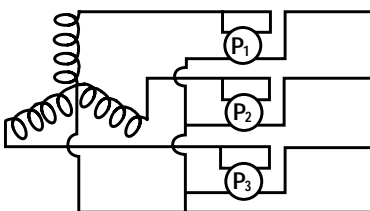


Fig. 10 Power in 3-phase 4-wire circuit, using three wattmeters.

In Fig. 8; total power is the algebraic sum of the two readings under all conditions of load and power factor. If the load is balanced, each instrument will read half the load; at 50% power factor one instrument reads all the load and the other reading is zero; at less than 50% power factor one reading will be negative. When the load is balanced, power may be measured with one wattmeter, using a "Y box" as shown in Fig. 9. This arrangement, which creates an artificial neutral, has two branches which have the same impedance and power factor as the wattmeters voltage circuit, which is the third branch of the Y. Total power is three times the reading of the wattmeter.

The 3-phase 4-wire circuits require three wattmeters as shown in Fig. 10. Total power is the algebraic sum of the three readings under all conditions of load and power factor. A 3-phase Y system with a grounded neutral is the equivalent of a 4-wire system and requires the use of three wattmeters. If the load is balanced, one wattmeter can be used with its current coil in series with one conductor and the voltage circuit connected between that conductor and the neutral. Total power is three times the wattmeter reading in this instrument.

These diagrams show the use of single-phase wattmeters. However, in practice, individual meters are not generally used because of possible difficulties in connection, problems in multiple readings, and the physical problem of arranging several meters.

Fortunately, polyphase instruments are available both in analog and digital configurations where the algebraic summation is achieved internally in the meter making the instrument reading the total measured power of the system.

### Wave Shape/Frequency

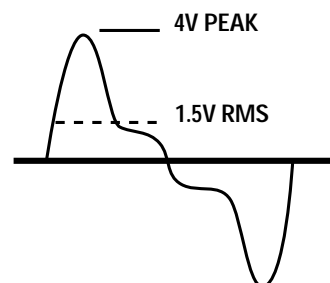
Wattmeters must use "RMS" sensing elements to sense volts and amps, or else power readings would be incorrect. These "RMS" sensing circuits have limitations in frequency response and type of wave shape that they can accept.

Frequency response is shown as the span of frequencies for a sine wave over which the instrument will operate. When dealing with non-sinusoidal wave forms such as a square wave, you must consider the harmonic content of the wave form. Thus, a 400 Hz non-sinusoidal wave shape with a large 3rd harmonic content requires a wattmeter capable of measuring at least 1200 Hz.

Wave shape capability is usually expressed as the "crest factor" for volts and amps. Crest factor is the ratio of the peak value to the RMS value of the wave shape.

#### Example for Determining Crest Factor:

$$\text{Crest Factor} = \frac{\text{Peak Value}}{\text{RMS Value}} = \frac{4}{1.5} = 2.6$$



A 3 phase 3-wire circuit requires two wattmeters connected as shown



## Electrical Power Measurements

### Crest Factor (Continued)

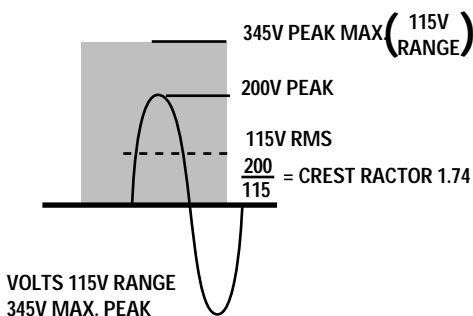
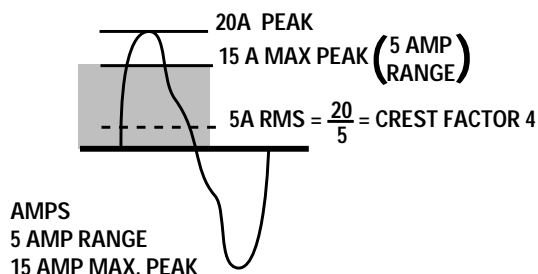
Electrodynamometer analog wattmeters can handle crest factors of 8 or more while the electronic type digital wattmeters are limited to a crest factor of about 3.

There is a way to use wattmeters successfully where the crest factor of the wave shape exceeds the limit of the instrument circuits. Take, for example, an instrument with a crest factor capability of 3 on both the current and voltage circuits. This means that if ranges of 5 amps RMS and 115 volts RMS are used, the limits are:

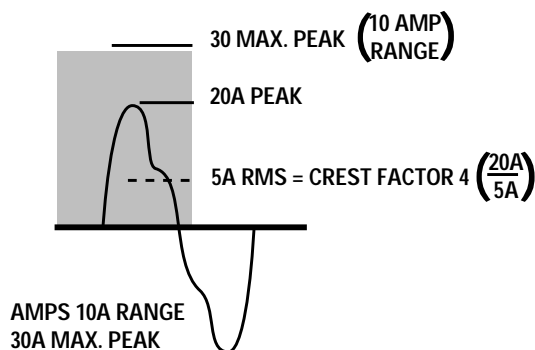
$$\text{Crest Factor} = \frac{15 \text{ amps peak}}{5 \text{ amps RMS}} = 3$$

$$\text{Crest Factor} = \frac{345 \text{ volts peak}}{115 \text{ volts RMS}} = 3$$

By examining the following inputs,



It can be seen that there is no problem with the voltage crest factor, but the current crest factor is 4, which exceeds the limit of 3. Most wattmeters have more than one range for the inputs, so the meter will no doubt have a 10 amp range. If the input is applied to the 10A and 115V range:



Now, even though the crest factor of the valve is still 4, it has been brought within the measuring range of the instrument by switching to a higher input range. Remember, the crest factor rating of a meter is fixed for each range. As long as the input does not exceed the peak value limit, a measurement can be made. A typical digital instrument would have the following limits:

### Amps

Input Range	Crest Factor Rating	Max. Amp Peak (70 Max.)
0.1A	3	0.3
0.3A	3	0.9
1.0A	3	3.0
3.0A	3	9.0
10.0A	3	30.0
30.0A	3	70.0

### Volts

Input Range	Crest Factor Rating	Max. Amp Peak (70 Max.)
3.0	3	9.0
10.0	3	30.0
30.0	3	90.0
100.0	3	300.0
300.0	3	900.0
600.0	3	1000.0